Math 70: 5.8 Division of Polynomials

Lesson 16

(Yes, we skipped 5.7)

5.8 Division of Polynomials

Objectives

1) Divide by a monomial
2) Divide by a polynomial
3) Use synthetic division to divide by a binomial
4) Know the limitations of synthetic division.

Long Division & Synthetic Division

Review Math 45 1) Divide polynomial by monomial

Review Math 45 2) Divide polynomial by polynomial using long division

* 3) Use synthetic division to divide a polynomial by a linear binomial with leading coefficient 1.

Divide

① $\dfrac{10x^3 - 5x^2 + 21x}{5x}$ ← polynomial / ← monomial (one term).

$= \dfrac{10x^3}{5x} - \dfrac{5x^2}{5x} + \dfrac{21x}{5x}$

write each term of the numerator as a separate fraction over the denominator

Signs between fractions come from the numerator.

$= \boxed{2x^2 - x + \dfrac{21}{5}}$

Reduce each fraction

② $\dfrac{3x^5y^2 - 15x^3y - x^2y - 6x}{2x^2y}$

$= \dfrac{3x^5y^2}{2x^2y} - \dfrac{15x^3y}{2x^2y} - \dfrac{x^2y}{2x^2y} - \dfrac{6x}{2x^2y}$

$= \boxed{\dfrac{3}{2}x^3y - \dfrac{15}{2}x - \dfrac{1}{2} - \dfrac{3}{xy}}$

CAUTION: The hardest part of these problems is remembering that this is the process!

Remember long division?

$$\begin{array}{r} 14 \\ 17\overline{)253} \\ -17 \\ \hline 83 \\ -68 \\ \hline 15 \end{array}$$

answer $14\frac{15}{17}$

$\frac{25}{17} = 1$ + stuff

$1 \times 17 = 17$

subtract $25 - 17$

— repeat —

$\frac{83}{17} = 4 +$

$17 \times 4 = 68$

Polynomial long division is exactly the same. (If you were taught to do the subtraction in your head, please don't do poly subtract in your head.)

Divide using a) long division
b) synthetic division

③ $\dfrac{2x^2 - x - 10}{x+2}$

$$
\begin{array}{r}
2x \\
x+2 \overline{)\;2x^2 - x - 10} \\
\underline{2x^2 + 4x }
\end{array}
$$

$\dfrac{2x^2}{x} = 2x$

Always Line up like terms!

$2x(x+2) = 2x^2 - 4x$

To subtract polynomials, we usually write:

$(2x^2 - x) - (2x^2 + 4x)$

dist negative;

$= 2x^2 - x - 2x^2 - 4x$

and combine like terms;

$= {}^-5x$

Now we're going to do this vertically by
- changing the signs
- adding like terms vertically

$$
\begin{array}{r}
2x - 5 \\
x+2 \overline{)\;2x^2 - x - 10} \\
\underline{-2x^2 - 4x } \\
-5x - 10 \\
\underline{+5x \;\; 10} \\
0
\end{array}
$$

Bring down -10 and repeat

answer: $\boxed{2x - 5}$

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(3) again, by synthetic division:

$$\begin{array}{r|lll} & & & \\ & & & \\ \hline & & & \end{array}$$

empty structure:
box, 2 blank lines,
line below.

$$\begin{array}{r|lll} -2 & & & \\ & & & \\ \hline & & & \end{array}$$

In the box, put the solution that corresponds to our divisor as a factor:

$x + 2 = 0$
$x = -2$

* changing the sign here takes care of all distribute-the-negative steps

$$\begin{array}{r|rrr} -2 & 2 & -1 & -10 \\ & & & \\ \hline & & & \end{array}$$

Across the first row, write the coefficients from the dividend, in standard form,
* using placeholder 0s for any missing terms

$$\begin{array}{r|rrr} -2 & 2 & -1 & -10 \\ & & & \\ \hline & 2 & & \end{array}$$

Bring down the 1st coefficient unchanged, write it below the line

$$\begin{array}{r|rrr} -2 & 2 & -1 & -10 \\ & & -4 & \\ \hline & 2 & & \end{array}$$

Multiply the number below the line by the number in the box, write result in next column, above the line

$$\begin{array}{r|rrr} -2 & 2 & -1 & -10 \\ & & -4 & \\ \hline & 2 & -5 & \end{array}$$

Add the numbers, put result below the line,

$$\begin{array}{r|rrr} -2 & 2 & -1 & -10 \\ & & -4 & 10 \\ \hline & 2 & -5 & 0 \end{array}$$

Repeat until the end of the line.

③ synthetic division, continued.

Notice that the degree of the dividend is reduced by one to get the quotient:

$$\begin{array}{r|rrr} -2 & 2x^2 & -1 & -10 \\ \hline & 2x & -5 & 0 \end{array}$$

The remainder appears in the last place

NOTICE: The numbers above the line appeared above each subtraction line in our long division process.
We don't need the first terms because they always add to zero.

CAUTION: Synthetic division only works in very limited circumstances

- The divisor must be linear
- The leading coefficient of the divisor must be 1.

NC ④ Divide $\dfrac{6x^2-19x+12}{3x-5}$

must use long division because leading coef $\neq 1$

$$\begin{array}{r} 2x \quad -3 \\ 3x-5 \enclose{longdiv}{6x^2-19x+12} \\ -\,6x^2+10x \quad\quad \\ \hline -9x+12 \\ +9x\mp 15 \\ \hline -3 \end{array}$$

$$\frac{6x^2}{3x} = 2x$$

$$\frac{-9x}{3x} = -3$$

answer $\boxed{2x-3+\dfrac{-3}{3x-5}}$

or $\boxed{2x-3-\dfrac{3}{3x-5}}$

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(4) Method 2: you can rewrite this so synthetic division can be used, but you must be comfortable with fractions!

$$\frac{6x^2 - 19x + 12}{3x - 5} \quad \frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)}$$

Want leading coef 1 in denom.

$$= \frac{\frac{6}{3}x^2 - \frac{19}{3}x + \frac{12}{3}}{\frac{3}{3}x - \frac{5}{3}}$$

$$= \frac{2x^2 - \frac{19}{3}x + 4}{x - \frac{5}{3}}$$

$$\begin{array}{c|ccc} \frac{5}{3} & 2 & -\frac{19}{3} & 4 \\ & & \frac{10}{3} & -5 \\ \hline & 2 & -3 & -1 \end{array}$$

GC: $\boxed{\text{MATH}}$ > Frac

which means $2x - 3 + \dfrac{-1}{x - 5/3}$ } cannot express remainder as a complex fraction

$$= 2x - 3 + \frac{-1\,(3)}{\left(x - \frac{5}{3}\right)(3)}$$

$$= \boxed{2x - 3 - \frac{3}{3x - 5}}$$

same as before

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no ⑤ Divide $\dfrac{7x^3+16x^2+2x}{x+4}$

linear, leading coef = 1

$$\begin{array}{r|rrrr} -4 & 7 & 16 & 2 & 0 \\ & & -28 & 48 & -200 \\ \hline & 7 & -12 & 50 & -20 \end{array}$$

$x+4=0$
solution -4 in box

answer $\boxed{7x^2-12x+50-\dfrac{20}{x+4}}$

yes ⑥ Divide $\dfrac{3x^4+2x^3-8x+6}{x^2-1}$

BE CAREFUL!
- Need a placeholder for x^2
- Be sure to line up like terms

$$\begin{array}{r l} & 3x^2+2x+3 \\ x^2-1 \,) & 3x^4+2x^3+0x^2-8x+6 \\ & -3x^4 \qquad\quad +3x^2 \\ \hline & 2x^3+3x^2-8x \\ & -2x^3 \qquad\quad +2x \\ \hline & 3x^2-6x+6 \\ & -3x^2 \qquad +3 \\ \hline & -6x+9 \end{array}$$

Can't use synthetic because it's not linear (divisor).

$\dfrac{3x^4}{x^2}=3x^2$

$\dfrac{2x^3}{x^2}=2x$

$\dfrac{3x^2}{x^2}=3$

answer: $\boxed{3x^2+2x+3+\dfrac{-6x+9}{x^2-1}}$

⑦ Divide $\dfrac{27x^3+8}{3x+2}$

Can't use synthetic because leading coef isn't 1.

$$\begin{array}{r l} & 9x^2-6x+4 \\ 3x+2 \,) & 27x^3+0x^2+0x+8 \\ & -27x^3 \mp 18x^2 \\ \hline & -18x^2+0x \\ & +18x^2+12x \\ \hline & 12x+8 \\ & -12x \mp 8 \\ \hline & 0 \end{array}$$

Need placeholders for both x^2 and x.

$\dfrac{27x^3}{3x}=9x^2$

$\dfrac{-18x^2}{3x}=-6x$

$\dfrac{12x}{3x}=4$

answer $\boxed{9x^2-6x+4}$

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(7) con't.

You can rewrite this problem so that synthetic division is possible, but you will need to be comfortable using fractions.

$$\frac{27x^3+8}{3x+2}\;\frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)}$$

← multiply by 1 (or divide all terms by 3)

$$= \frac{\frac{27}{3}x^3 + \frac{8}{3}}{\frac{3x}{3} + \frac{2}{3}}$$

$$= \frac{9x^3 + \frac{8}{3}}{x + \frac{2}{3}}$$

← now denom is linear with leading coefficient 1.

$$\begin{array}{r|rrrr} -\frac{2}{3} & 9 & 0 & 0 & \frac{8}{3} \\ & & -6 & 4 & -8/3 \\ \hline & 9 & -6 & 4 & 0 \end{array}$$

← MATH>FRAC is your friend

$$\boxed{9x^2 - 6x + 4}$$

same as we got before

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Note (7) was a way to remember the trinomial factor for the sum of cubes...

(8) Divide $\dfrac{2x^3 - x^2 - 13x + 1}{x - 3}$

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ & & 6 & 15 & 6 \\ \hline & 2 & 5 & 2 & 7 \end{array}$$

answer $\boxed{2x^2 + 5x + 2 + \dfrac{7}{x-3}}$

(9) Divide $\dfrac{x^4 - 2x^3 - 11x^2 + 5x + 34}{x + 2}$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -11 & 5 & 34 \\ & & -2 & 8 & 6 & -22 \\ \hline & 1 & -4 & -3 & 11 & 12 \end{array}$$

answer $\boxed{x^3 - 4x^2 - 3x + 11 + \dfrac{12}{x+2}}$

(10) $\dfrac{3x^2 - 4}{x - 1}$

$$\begin{array}{r|rrr} 1 & 3 & 0 & -4 \\ & & 3 & 3 \\ \hline & 3 & 3 & -1 \end{array}$$

Don't forget placeholder!

answer $\boxed{3x + 3 + \dfrac{-1}{x-1}}$

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(11) $$\frac{x^4 - \frac{2}{3}x^3 + x}{x - 3}$$

$$\begin{array}{r|rrrrr} 3 & 1 & -2/3 & 0 & 1 & 0 \\ & & 3 & 7 & 21 & 66 \\ \hline & 1 & 7/3 & 7 & 22 & 66 \end{array}$$

Need placeholders for both x^2 and constant terms

Use GC with >frac!

answer $\boxed{x^3 + \frac{7}{3}x^2 + 7x + 22 + \frac{66}{x-3}}$

(12) $$\frac{2x^5 + x^3 + 3 - 6x^4 - 4x}{x^2 - 3}$$

out of order! and x^2 needs placeholder!

can't use synthetic because it's not linear

$$\begin{array}{r l} & 2x^3 - 6x^2 + 7x - 18 \\ x^2 - 3 \,\big) & 2x^5 - 6x^4 + x^3 + 0x^2 - 4x + 3 \\ & -2x^5 \qquad\quad + 6x^3 \\ \hline & -6x^4 + 7x^3 + 0x^2 \\ & +6x^4 \qquad\quad \mp 18x^2 \\ \hline & 7x^3 - 18x^2 - 4x \\ & -7x^3 \qquad\quad +21x \\ \hline & -18x^2 + 17x + 3 \\ & +18x^2 \qquad\quad \mp 54 \\ \hline & 17x - 51 \end{array}$$

$\frac{2x^5}{x^2} = 2x^3$

$\frac{-6x^4}{x^2} = -6x^2$

$\frac{7x^3}{x^2} = 7x$

$\frac{-18x^2}{x^2} = -18$

answer: $\boxed{2x^3 - 6x^2 + 7x - 18 + \frac{17x - 51}{x^2 - 3}}$

6.4.87 **Divide.**

$$(14x^4 - 4x^2 + 21x^3 - 6x) \div (14x + 21)$$

$$(14x^4 - 4x^2 + 21x^3 - 6x) \div (14x + 21) = x^3 + -\frac{2}{7}x$$

(Simplify your answer. Do not factor.)

"Simplify your answer" means "reduce all fractions."

"Do not factor" means "Do not do this problem by factoring and canceling."

Should be done by long division, or with adjustment, by synthetic division.

Long Division:

$$\begin{array}{r} x^3 \qquad\qquad - \frac{2}{7}x \qquad\qquad \\ 14x + 21 \,\overline{)\, 14x^4 + 21x^3 - 4x^2 - 6x + 0} \\ \mp 14x^3 \mp 21x^3 \qquad\qquad\qquad\qquad \\ \hline 0 - 4x^2 - 6x \qquad \\ \pm 4x^2 \pm 6x \qquad \\ \hline 0 + 0. \end{array}$$

$$14x(?) = -4x^2$$
$$(?) = \frac{-4x^2}{14x} = -\frac{2}{7}x$$

solution: $\boxed{x^3 - \frac{2}{7}x}$

Synthetic Division:

$$\frac{\frac{1}{14}(14x^4 + 21x^3 - 4x^2 - 6x + 0)}{\frac{1}{14}(14x + 21)}$$

← need coefficient of x to be 1. Use GC ▷frac to do multiplying.

$$= \frac{x^4 + \frac{3}{2}x^3 - \frac{2}{7}x^2 - \frac{3}{7}x + 0}{x + \frac{3}{2}}$$

← $x + \frac{3}{2} = x - \left(-\frac{3}{2}\right)$

Put $-\frac{3}{2}$ in box.

x^4 ↓

$$\begin{array}{r|rrrrr} -\frac{3}{2} & 1 & \frac{3}{2} & -\frac{2}{7} & -\frac{3}{7} & 0 \\ & & -\frac{3}{2} & 0 & +\frac{3}{7} & 0 \\ \hline & 1 & 0 & -\frac{2}{7} & 0 & 0 \end{array}$$

↑ x^3 one degree less.

solution $\boxed{x^3 - \frac{2}{7}x}$

A different problem. (Notice the instructions)

Simplify

$$\frac{14x^4 + 21x^3 - 4x^2 - 6x}{14x + 21}$$

$$= \frac{7x^3(2x+3) - 2x(2x+3)}{7(2x+3)}$$

$$= \frac{(7x^3 - 2x)\cancel{(2x+3)}}{7\cancel{(2x+3)}}$$

$$= \boxed{\frac{x(7x^2 - 2)}{7}}$$